# An Identification Strategy for Proxy-SVARs with Weak Proxies

Luca Fanelli

Joint with Giovanni Angelini and Giuseppe Cavaliere
University of Bologna

SIdE Webinar
8 March 2021

# Motivations

- **Proxy-SVARs (SVAR-IVs)** have become standard tools to track dynamic causal effects **[IRFs]** in empirical macroeconomics: Mertens and Ravn (2013 AER); Stock and Watson (2018 EJ).
- **Main challenge:** external variables/proxies/instruments can be **weak**, i.e. poorly correlated with the **target structural shocks,** $\varepsilon_{1,t}$.
  $Cov(z_t, \varepsilon_{1,t}) = \frac{C}{T^{1/2}}$ à la Staiger and Stock (1997, ECMA).
- **Proxy-SVARs with weak proxies**: similar to IV regressions with weak instruments
  $\Rightarrow$ asymptotic inference **is non-standard**.

# Proxy-SVARs with weak proxies

- **Frequentist framework**:
  Montiel-Olea, Stock and Watson (2020, JoE) **extend the logic of Anderson-Rubin tests** to proxy-SVARs.
- **Bayesian framework**:
  Giacomini, Kitagawa and Read (2020, JoE), set-identification approach.

# Proxy-SVARs with weak proxies (cont'd)

- Montiel-Olea, Stock and Watson (2020, JoE) approach:
- **Manageable** with **one target shock**, i.e. $\varepsilon_{1,t}$ scalar, $k = 1$.
- **Problematic** with **multiple target shocks**, i.e. $\varepsilon_{1,t}$ is $k \times 1$, $k > 1$, e.g. $\varepsilon_{1,t} := (\varepsilon_t^{tax}, \varepsilon_t^{g})'$:
  - at least $k^2 > \frac{1}{2}k(k-1)$ restrictions needed to build S-regions;
  - **conservative** inference.

# How we attack the problem

- Standard '**direct**' approach:

$$\varepsilon_t := \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \begin{matrix} k \times 1 & \textbf{target shocks} \\ (n-k) \times 1 & \text{auxiliary shocks} \end{matrix}$$

proxies: $z_t = \Phi \varepsilon_{1,t} + \omega_{z,t}$ , $\omega_{z,t} \perp \varepsilon_t$ , $rank[\Phi_0] =?$

- We might have proxies $v_t$ for $s \leq n-k$ **auxiliary shocks** $\tilde{\varepsilon}_{2,t}$:

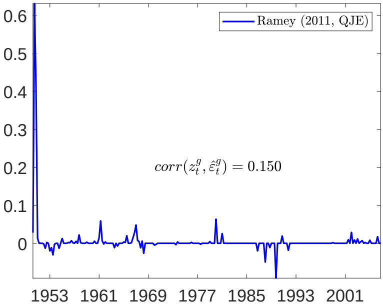
$$v_t = \Lambda \tilde{\varepsilon}_{2,t} + \omega_{v,t} \ , \ \omega_{v,t} \perp \varepsilon_t \ , \ rank[\Lambda_0] = s$$

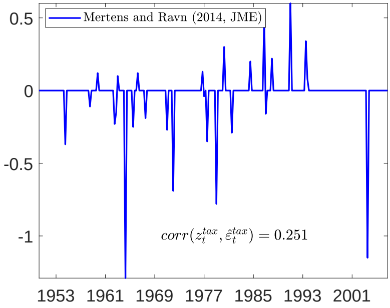
- **Objective:** use **strong** proxies $v_t$ for $\tilde{\varepsilon}_{2,t}$ **to recover the dynamic causal effects of the target shocks** $\varepsilon_{1,t}$.
- **Advantage: circumvent non-standard methods of inference** when $z_t$ is suspected to be **weak** for $\varepsilon_{1,t}$, especially when **k>1**.

# Contributions

- **Identification strategy: indirect approach**
  - **B-form** $\longleftrightarrow$ **A-form** representation of proxy-SVAR and design a **MD estimation** approach based on the A-form
  - derive **necessary and sufficient rank condition** for identification
  - $\Rightarrow$ **standard asymptotics** if $v_t$ is **strong** for $\tilde{\varepsilon}_{2,t}$.
- **Diagnostic test** for **instrument relevance** $v_t$ **based on bootstrap resampling - MBB** $\Rightarrow$ **no pre-testing issues**.
- **Empirical**: focus in the **size and uncertainty** of US fiscal multipliers from a fiscal proxy-SVAR.
  Ideal case, **k=2**, and narrative proxies for fiscal shocks suspected to be **weak**.

# Fiscal proxies



# Outline

- Identification strategy
  - B-form and A-form of proxy-SVARs
  - Indirect approach: MD estimation + asymptotic results
  - Example: tracking fiscal multipliers
- Bootstrap diagnostic test
  - Sketch of the idea
  - Mechanics (part)
  - Operational form
  - Monte Carlo
  - Screening on $\tau_v^*$
- Fiscal multipliers from a quarterly fiscal proxy-SVAR
- Concluding remarks

# IDENTIFICATION STRATEGY

# B-form in proxy-SVARs

- SVAR: $Y_t - \Pi_1 Y_{t-1} = u_t \ , \ \ u_t = B\varepsilon_t \ , \mathrm{E}(\varepsilon_t \varepsilon_t') = I_n$
- Proxy-SVAR:

$$\varepsilon_t := \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \quad \begin{matrix} \textbf{target shocks} & k \times 1 \\ \text{auxiliary shocks} & (n-k) \times 1 \end{matrix}$$

$$u_t = B\varepsilon_t = B_1 \underbrace{\varepsilon_{1,t}}_{\text{target}} + \ B_2 \ \underbrace{\varepsilon_{2,t}}_{\text{auxiliary}}$$

- **Target:** $IRF_{i,j}(h) = \gamma_{i,j}(h) = e_i' \left(\Pi_1\right)^h B_1 e_j, \ \ j = 1, ..., \mathbf{k}$

# A-form in proxy-SVARs

- A-form:

$$Au_t = \varepsilon_t \equiv \begin{cases} A_1' u_t = \varepsilon_{1,t} & k \times 1 \\ A_2' u_t = \varepsilon_{2,t} & (n-k) \times 1 \end{cases}$$

- focus on the $k$ structural equations:

$$A_1' u_t = (A_{11}' \vdots A_{12}') \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix} = A_{11}' u_{1,t} + A_{12}' u_{2,t} = \varepsilon_{1,t}$$

- **link:** $B_1 = \Sigma_u A_1 \Rightarrow \gamma_{i,j}(h) = e_i' (\Pi_1)^h B_1 e_j, \ \ j = 1, ..., \mathbf{k}$
- **Auxiliary shocks have a role in the identification of** $A_1$ **and in recovering the target shocks !**

# Strategy induced by A-form

- Take:

$$A'_{11}u_{1,t} + A'_{12}u_{2,t} = \varepsilon_{1,t}$$

- Assume that $A'_{11}$ $(k \times k)$ is invertible (**not needed in our framework**):

$$u_{1,t} = \Psi u_{2,t} + \left(A'_{11}\right)^{-1}\varepsilon_{1,t} \quad , \quad \Psi := -\left(A'_{11}\right)^{-1}A'_{12}$$

  and recall that

$$u_{2,t} = B_{21}\varepsilon_{1,t} + B_{22}\varepsilon_{2,t}$$

- $\Rightarrow$ we might consider proxies $v_t$ for the auxiliary shocks $\varepsilon_{2,t}$ !
- With $k > 1$, $A'_{11}$ must be triangular: $\frac{1}{2}k(k-1)$ restrictions in Mertens and Ravn's (2013, AER) in their IV approach.
  **Not necessary in our framework.**

# Example 1: macro and financial uncertainty shocks

- Ludvigson, Ma and Ng (2019, AEJ-Macro); Angelini and Fanelli (2019, JAE):

$$Y_t := \begin{pmatrix} U_t^F \\ U_t^M \\ a_t \end{pmatrix}, \quad \varepsilon_t := \begin{pmatrix} \varepsilon_t^F \\ \varepsilon_t^M \\ \varepsilon_t^a \end{pmatrix} \begin{matrix} \text{financial uncertainty shock} \\ \text{macro uncertainty shock} \\ \text{real economic activity shock} \end{matrix}$$

- $\varepsilon_{1,t} := (\varepsilon_t^F, \varepsilon_t^M)'$, $k = 2$ **target shocks,** B-form:

$$\begin{pmatrix} u_t^F \\ u_t^M \\ u_t^a \end{pmatrix} = \begin{pmatrix} \beta_{F,F} & \beta_{F,M} \\ \beta_{M,F} & \beta_{M,M} \\ \beta_{a,F} & \beta_{a,M} \end{pmatrix} \begin{pmatrix} \varepsilon_t^F \\ \varepsilon_t^M \end{pmatrix} + B_2 \varepsilon_t^a$$

- At least 2 proxies + 1 restriction

$$\begin{pmatrix} z_t^F \\ z_t^M \end{pmatrix} = \Phi \begin{pmatrix} \varepsilon_t^F \\ \varepsilon_t^M \end{pmatrix} + \omega_{z,t}$$

# Example 1: uncertainty shocks (cont'd)

- A-form, $A_1' u_t = \varepsilon_{1,t}$ :

$$\underbrace{\begin{pmatrix} \alpha_{F,F} & \alpha_{F,M} \\ \alpha_{M,F} & \alpha_{M,M} \end{pmatrix}}_{A_{11}'} \underbrace{\begin{pmatrix} u_t^F \\ u_t^M \end{pmatrix}}_{u_{1,t}} + \underbrace{\begin{pmatrix} \alpha_{F,a} \\ \alpha_{M,a} \end{pmatrix}}_{A_{12}'} u_t^a = \underbrace{\begin{pmatrix} \varepsilon_t^F \\ \varepsilon_t^M \end{pmatrix}}_{\varepsilon_{1,t}}$$

- At least one proxy ($s \leq n - k = 1$) + 1 restriction: e.g. $\alpha_{F,a} = 0$ (or $\alpha_{F,M} = 0$)

$$v_t = \lambda \varepsilon_t^a + \omega_{v,t}$$

- **implied moment conditions**

$$A_1' \Sigma_u A_1 = I_2 \quad \text{, 3 moment conditions}$$

$$A_1' \Sigma_{u,v} = 0_{2\times 1}, \quad \text{2 moment conditions}$$

# Example 2: Kilian's supply oil shock

- Kilian (2009, AER):

$$Y_t := \begin{pmatrix} prod_t \\ rea_t \\ rpo_t \end{pmatrix} \begin{matrix} \text{global crude oil production} \\ \text{global real economic activity} \\ \text{real oil prices} \end{matrix} \quad \varepsilon_t := \begin{pmatrix} \varepsilon_t^{Supply} \\ \varepsilon_t^{Ag.Dem} \\ \varepsilon_t^{Oil.Spec.Dem} \end{pmatrix}$$

- $\varepsilon_{1,t} := \varepsilon_t^{Supply}$, $k = 1$ **target shock**

$$\begin{pmatrix} u_t^{prod} \\ u_t^{rea} \\ u_t^{rpo} \end{pmatrix} = \begin{pmatrix} \beta_{1,1} \\ \beta_{2,1} \\ \beta_{3,1} \end{pmatrix} \underbrace{\left(\varepsilon_t^{Supply}\right)}_{\text{target shock}} + B_2 \begin{pmatrix} \varepsilon_t^{Ag.Demand} \\ \varepsilon_t^{Oil.Spec.Dem} \end{pmatrix}$$

- Kilian's (2008, REStat) proxy: $z_t^{Oil} = \phi\varepsilon_t^{Supply} + \omega_{z,t}$
- **Weak** in Montiel-Olea, Stock and Watson (2020, JoE) robust first-stage F-test = **9.4**

# Example 2: Kilian's supply oil shock (cont'd)

- A-form:

$$\alpha_{1,1} u_t^{prod} + \begin{pmatrix} \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} u_t^{rea} \\ u_t^{rpo} \end{pmatrix} = \underbrace{\varepsilon_t^{Supply}}_{\varepsilon_{1,t}}$$

- A least two proxies, $s = n - k = 2$:

$$\begin{pmatrix} v_t^{rea} \\ v_t^{rpo} \end{pmatrix} = \Lambda \begin{pmatrix} \varepsilon_t^{Ag.Demand} \\ \varepsilon_t^{Oil.Spec.Dem} \end{pmatrix} + \omega_{v,t}$$

$v_t^{rea}$ ≡Ravazzolo and Vespignani's (2020, CJoE) index;
$v_t^{rpo}$ ≡"future on Brent or WTI".

- **implied moment conditions**

$$A_1' \Sigma_u A_1 = I_1 \text{ , 1 moment condition}$$

$$A_1' \Sigma_{u,v} = 0_{2\times 1}, \quad \text{2 moment conditions}$$

# Indirect-MD approach: assumptions and definition

- ASSUMPTION 1: DGP∈VAR, $\Pi_1$ stable;
- ASSUMPTION 2: $u_t \sim \alpha$-mixing;
- ASSUMPTION 3: $A = B^{-1} \Rightarrow rank[A_1'] = k = rank[B_1]$
- ASSUMPTION 4: given $\tilde{\varepsilon}_{2,t}$ subset of $\varepsilon_{2,t}$, $dim(\tilde{\varepsilon}_{2,t}) = s \leq n - k$

$$v_t = \Lambda \tilde{\varepsilon}_{2,t} + \omega_{v,t} \quad , \; \omega_{v,t} \perp \varepsilon_t$$

- DEFINITION: $v_t$ is:
  - **strong** for $\tilde{\varepsilon}_{2,t}$ if $rank[\Lambda] = s$ in $\mathcal{N}_{\lambda_0}$ regardless of $T$;
  - **weak** for $\tilde{\varepsilon}_{2,t}$ if $\Lambda = \Lambda_1 + T^{-1/2}C$, $C \neq 0_{s\times s}$, $rank[\Lambda_1] < s$ in $\mathcal{N}_{\lambda_0}$.

# Indirect-MD approach: moment conditions

- Recall: **estimate** $\gamma_{i,j}(h) = e_i' (\Pi_1)^h B_1 e_j$, $j = 1, ..., \mathbf{k}$ by **exploiting** $B_1 = \Sigma_u A_1$
- A-form:

$$A_1' u_t = \varepsilon_{1,t}$$

- Under Assumption 4, implied **moment conditions:**

$$\begin{array}{ll} A_1'\Sigma_u A_1 = I_k & \frac{1}{2}k(k+1) \times 1 \\ A_1'\Sigma_{u,v} = 0_{ks\times 1} & ks \times 1 \end{array}$$

- $k > 1$, at least $\frac{1}{2}k(k-1)$ restrictions on $A_1'$ :

$$vec(A_1') = S_{A_1}\alpha + s_{A_1}$$

# Indirect-MD approach: distance function

- Summarize moment conditions and restrictions in the **distance function**:

$$g(\sigma_0^+, \alpha_0) := \text{vec}\begin{pmatrix} A_1'\Sigma_u A_1 - I_k \\ A_1'\Sigma_{u,v} \end{pmatrix} = 0_{m\times 1},\ A_1 := A_1(\alpha).$$

- $\hat{\sigma}_T^+ := (\text{vec}(\hat{\Sigma}_{u,v})', \text{vech}(\hat{\Sigma}_u)')'$ easily recoverable from

$$\begin{pmatrix} Y_t \\ v_t \end{pmatrix} = \begin{pmatrix} \Pi_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Y_{t-1} \\ v_{t-1} \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}$$

$$\Sigma_\eta := \mathrm{E}\left\{\begin{pmatrix} u_t \\ v_t \end{pmatrix}(u_t', v_t')\right\} = \begin{pmatrix} \Sigma_u & \Sigma_{u,v} \\ \Sigma_{v,u} & \Sigma_v \end{pmatrix}$$

**consistent** and **asymp. Gaussian** under Ass. 1-2 and 4.

# Indirect-MD approach: estimation

- Given $g(\sigma_0^+, \alpha_0) = 0_{m\times 1}$
- Then

$$\hat{\alpha}_T := \arg\min_{\alpha\in\mathcal{T}_\alpha} \hat{g}_T(\hat{\sigma}_T^+, \alpha)' \hat{V}_{gg}(\bar{\alpha})^{-1} \hat{g}_T(\hat{\sigma}_T^+, \alpha)$$

  $\hat{V}_{gg}(\bar{\alpha}) := G_{\sigma^+}(\sigma^+, \alpha)\hat{V}_{\sigma^+}G_{\sigma^+}(\sigma^+, \alpha)'$,
  $G_{\sigma^+}(\sigma^+, \alpha) := \frac{\partial g(\sigma^+, \alpha)}{\partial \sigma^{+\prime}}$ Jacobian,
  $V_{\sigma^+} := \lim_{T\to\infty} Var(T^{1/2}(\hat{\sigma}_T^+ - \sigma_0^+))$
  $\bar{\alpha}$ preliminary estimate of $\alpha$

- Newey and McFadden (1994, Handbook, Ch.36)

# Indirect-MD approach: identification

PROPOSITION 1 [RANK CONDITION] Under Assumptions 1-4, the MD objective function has **unique** minimum in $\mathcal{N}_{\alpha_0}$ **iff**:

$$rank\left[\begin{pmatrix} 2D_k^{+}(A_1{}'\Sigma_u \otimes I_k) \\ (\Sigma_{v,u} \otimes I_k) \end{pmatrix} S_{A_1}\right] = a,\ a := \dim(\alpha).$$

**Necessary order** condition:
$\ell \geq \frac{1}{2}k(k-1) \Leftrightarrow a \leq \frac{1}{2}k(k+1) + ks$

COROLLARY 1: If rank condition holds $\Rightarrow rank[\Lambda_0] = s$ in $\mathcal{N}_{\lambda_0}$, i.e. $v_t$ **strong** for $\tilde{\varepsilon}_{2,t}$.

# Indirect-MD approach: asymptotic properties

PROPOSITION 2. Under Assumptions 1-4, **rank condition holds**, $\alpha_0 \in int(\mathcal{T}_\alpha)$, as $T \longrightarrow \infty$:

(i) $(\hat{\alpha}_T - \alpha_0) \xrightarrow{p} 0_{a \times 1}$

(ii) $T^{1/2} (\hat{\alpha}_T - \alpha_0) \xrightarrow{d} \mathcal{N}(0, V_\alpha)$, $V_\alpha := \left\{ G_\alpha{}' V_{gg}(\bar{\alpha})^{-1} G_\alpha \right\}^{-1}$

- Implications on IRFs: because of $B_1 = \Sigma_u A_1$

$$T^{1/2} \left( \widehat{\gamma}_{i,j}(h) - \gamma_{i,j,0}(h) \right) \xrightarrow{d} \mathcal{N}(0, V_{\gamma_{ij}}), \ h = 0, ...., h_{\max}$$

- **Inference is standard**

# Indirect-MD approach: qualifications on the methodology

- **A-form** of proxy-SVARs: typical of the Bayesian approach: Caldara and Herbst (2019, AEJ-Macro), Arias, Rubio-Ramirez and Waggoner (2020, JoE), Giacomini, Kitagawa and Read (2020, JoE):
  - **proxies are used to instrument** the **target shocks**, not the **auxiliary** ones
- Caldara and Kamps (2017, REStud), fiscal proxy-SVAR, use **non-fiscal proxies** to instrument **non-fiscal shocks**, **Bayesian - penalty function - approach.**
- Our: **frequentist approach, point-identification, does not necessarily require** a policy reaction function.
- When **k>1 MD** estimation more general than **IV**, does not require **Choleski-type restrictions**, see e.g. Mertens and Ravn (2013, AER).

# Example: tracking fiscal multipliers from a fiscal proxy-SVAR

- VAR: $n = 4$

$$Y_t = \begin{pmatrix} TAX_t \\ G_t \\ DGP_t \\ RR_t \end{pmatrix} \begin{array}{l} \text{tax revenues} \\ \text{fiscal spending} \\ \text{real GDP} \\ \text{real interest rate} \end{array}$$

- **Fiscal multipliers**: $k = 2$, $h = 0, 1, \ldots$

$$\mathbb{M}_{h,tax} := \frac{\left(\frac{\partial DGP_{t+h}}{\partial \varepsilon_t^{tax}}\right)}{\beta_{tax,tax}} \times \text{scale} \;, \quad \mathbb{M}_{h,g} := \frac{\left(\frac{\partial DGP_{t+h}}{\partial \varepsilon_t^{g}}\right)}{\beta_{g,g}} \times \text{scale}$$

Blanchard and Perotti (2002, QJE);
Mertens and Ravn (2014, JME);
Caldara and Kamps (2017, REStud).

# Example: fiscal multipliers, B-form, direct approach

- B-form: $k = 2$, $(n - k = 2)$

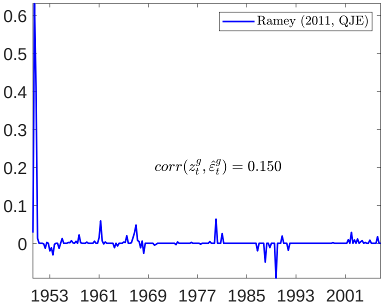
$$\begin{pmatrix} u_t^{tax} \\ u_t^{g} \\ u_t^{y} \\ u_t^{rr} \end{pmatrix} = \underbrace{\begin{pmatrix} \beta_{tax,tax} & \beta_{tax,g} \\ \beta_{g,tax} & \beta_{g,g} \\ \beta_{y,tax} & \beta_{y,g} \\ \beta_{rr,tax} & \beta_{rr,g} \end{pmatrix}}_{B_1} \underbrace{\begin{pmatrix} \varepsilon_t^{tax} \\ \varepsilon_t^{g} \end{pmatrix}}_{\text{target } \varepsilon_{1,t}} + B_2 \underbrace{\begin{pmatrix} \varepsilon_t^{y} \\ \varepsilon_t^{mp} \end{pmatrix}}_{\text{auxiliary } \varepsilon_{2,t}}$$

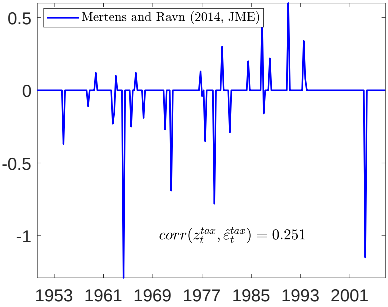
- Proxies:

$$\underbrace{\begin{pmatrix} z_t^{tax} \\ z_t^{g} \end{pmatrix}}_{z_t} = \underbrace{\begin{pmatrix} \varphi_{11} & 0 \\ 0 & \varphi_{22} \end{pmatrix}}_{\Phi} \underbrace{\begin{pmatrix} \varepsilon_t^{tax} \\ \varepsilon_t^{g} \end{pmatrix}}_{\varepsilon_{1,t}} + \underbrace{\begin{pmatrix} \omega_t^{tax} \\ \omega_t^{g} \end{pmatrix}}_{\omega_{z,t}}$$

- $\mathrm{E}(u_t z_t') = \Sigma_{u,z} = B_1 \Phi'$.

# Fiscal proxies



# Multipliers

## 68%-MBB CI and projected S-region

direct

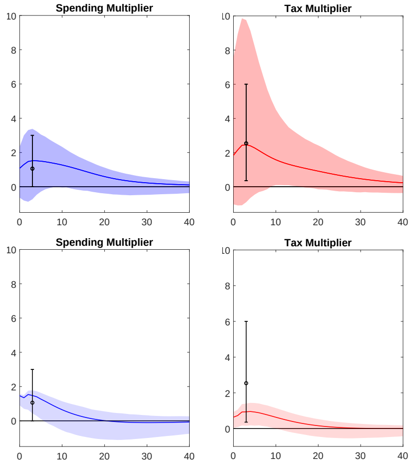
| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 1.08 | 1.84 |
| (−0.64;2.34) | (−1.03;7.58) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.50[3] | 2.46[3] |
| (−0.73;3.38) | (−0.91;9.76) |

direct A&R projected

| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 0.74 | 1.91 |
| (0.00;3.00) | (0.22;6.00) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.06[3] | 2.55[3] |
| (0.00;3.00) | (0.37;6.00) |

indirect-MD

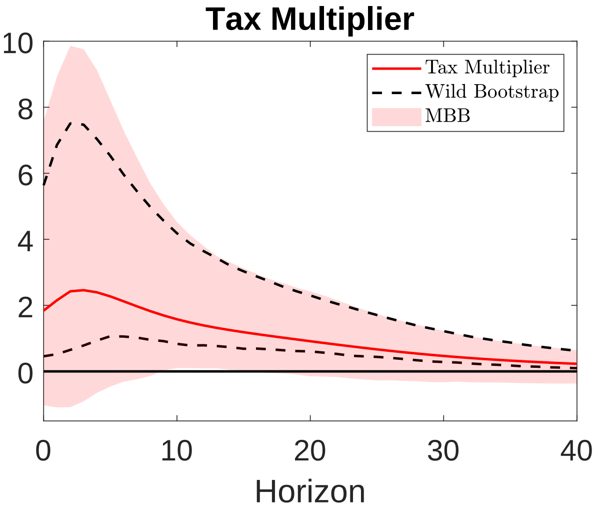
| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 1.47 | 0.64 |
| (0.90;1.56) | (0.04;0.93) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.54[2] | 0.96[4] |
| (0.64;1.76) | (0.18;1.44) |




| $p$-val |
|---|
| 0.003 |

| $p$-val |
|---|
| 0.822 |

# MBB vs Wild Bootstrap



# Example: Identification-robust method

- Proxies $z_t$ are suspected to be **weak**:

$$\Sigma_{u,z} = B_1\Phi' \Rightarrow \begin{matrix} k \times k \\ k \times (n-k) \end{matrix} \begin{pmatrix} \Sigma_{u1,z} \\ \Sigma_{u2,z} \end{pmatrix} = \begin{pmatrix} B_{11}\Phi' \\ B_{21}\Phi' \end{pmatrix}$$

- With $B_{11} = I_k$ ($k^2 = 4$ restrictions)
- solve $\Phi$ (regardless of rank) as function of reduced form parameters:
- $\Rightarrow \Phi' = \Sigma_{u1,z} \Rightarrow \Sigma_{u2,z} - B_{21}\Sigma_{u1,z} = 0_{k(n-k)}$.

# Example: Identification-robust method (cont'd)

- Null hypothesis (at given horizon $h$)

$$H_0 : \left( \frac{\partial GDP_{t+h}}{\partial \varepsilon_t^{tax}} , \frac{\partial GDP_{t+h}}{\partial \varepsilon_t^{g}} \right) = e_3'(\Pi_1)^h B_1 = (a_{tax,}^h, b_g^h)$$

implies

$$e_3'(\Pi_1)^h \Sigma_{u,z} - (a_{tax,}^h, b_g^h)(I_k \vdots 0)\Sigma_{u,z} = (0,0).$$

- For $\delta := (vec(\Pi_1)', vec(\Sigma_{u,z})')'$ and $T^{1/2}(\hat{\delta}_T - \delta_0) \xrightarrow{d} \mathcal{N}(0, V_\delta)$

$$\Rightarrow \; f(\delta, a_{tax,}^h, b_g^h) = 0_{2\times 1} \quad ; \quad T^{1/2} f(\hat{\delta}_T, a_{tax,}^h, b_g^h) \xrightarrow{d} \mathcal{N}(0_{2\times 1}, V_f)$$

- **S-region** with asymptotic coverage $\geq 1 - \varrho$:

$$\left\{ (a_{tax,}^h, b_g^h), \; Tf(\hat{\delta}_T, a_{tax,}^h, b_g^h)' \left(\hat{V}_f\right)^{-1} f(\hat{\delta}_T, a_{tax,}^h, b_g^h) \leq \chi^2_{2,1-\varrho} \right\}$$

# Multipliers

## 68%-MBB CI and projected S-region

direct

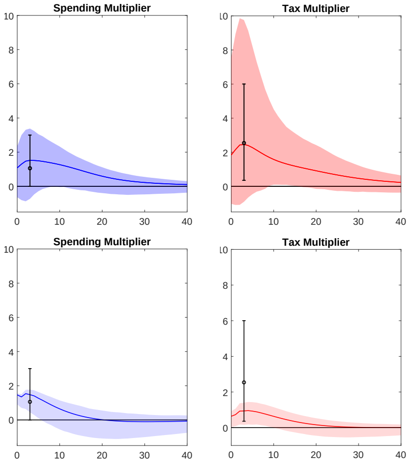
| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 1.08 | 1.84 |
| $(-0.64;2.34)$ | $(-1.03;7.58)$ |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.50[3] | 2.46[3] |
| $(-0.73;3.38)$ | $(-0.91;9.76)$ |

direct A&R projected

| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 0.74 | 1.91 |
| (0.00;3.00) | (0.22;6.00) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.06[3] | 2.55[3] |
| (0.00;3.00) | (0.37;6.00) |

indirect-MD

| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 1.47 | 0.64 |
| (0.90;1.56) | (0.04;0.93) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.54[2] | 0.96[4] |
| (0.64;1.76) | (0.18;1.44) |




| $p$-val |
|---|
| 0.003 |

| $p$-val |
|---|
| 0.822 |

# Example: fiscal multipliers, A-form, indirect approach

- A-form $k = 2$, $n - k = 2$

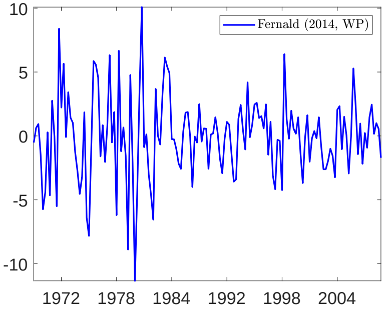
$$\underbrace{\begin{pmatrix} \alpha_{tax,tax} & \alpha_{tax,g} \\ \alpha_{g,tax} & \alpha_{g,g} \end{pmatrix}}_{A'_{11}} \underbrace{\begin{pmatrix} u_t^{tax} \\ u_t^{g} \end{pmatrix}}_{u_{1,t}}$$

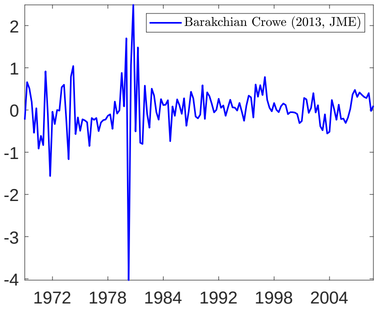
$$+ \underbrace{\begin{pmatrix} \alpha_{tax,y} & \alpha_{tax,r} \\ \alpha_{g,y} & \alpha_{g,r} \end{pmatrix}}_{A'_{12}} \underbrace{\begin{pmatrix} u_t^{y} \\ u_t^{rr} \end{pmatrix}}_{u_{2,t}} = \underbrace{\begin{pmatrix} \varepsilon_t^{tax} \\ \varepsilon_t^{g} \end{pmatrix}}_{\varepsilon_{1,t}}$$

- $\frac{1}{2}k(k-1) = 1$ restrictions on $A'_1 \equiv (A'_{11} \vdots A'_{12})$ **consistent with rank condition**
- Proxies:

$$\begin{pmatrix} v_t^{tfp} \\ v_t^{mp} \end{pmatrix} = \Lambda \begin{pmatrix} \varepsilon_t^{y} \\ \varepsilon_t^{mp} \end{pmatrix} + \begin{pmatrix} \omega_t^{tfp} \\ \omega_t^{mp} \end{pmatrix}.$$

# Indirect-MD: non-fiscal proxies for auxiliary shocks



# Multipliers

## 68%-MBB CI and projected S-region

direct

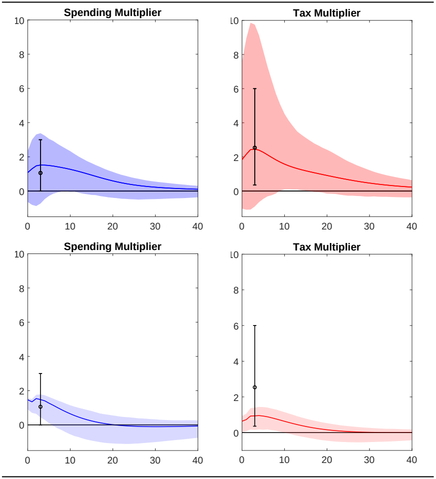
| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 1.08 | 1.84 |
| $(-0.64;2.34)$ | $(-1.03;7.58)$ |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.50[3] | 2.46[3] |
| $(-0.73;3.38)$ | $(-0.91;9.76)$ |

direct A&R projected

| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 0.74 | 1.91 |
| (0.00;3.00) | (0.22;6.00) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.06[3] | 2.55[3] |
| (0.00;3.00) | (0.37;6.00) |

indirect-MD

| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 1.47 | 0.64 |
| (0.90;1.56) | (0.04;0.93) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.54[2] | 0.96[4] |
| (0.64;1.76) | (0.18;1.44) |




| $p$-val |
|---|
| 0.003 |

| $p$-val |
|---|
| 0.822 |

# BOOTSTRAP DIAGNOSTIC TEST FOR INSTRUMENT RELEVANCE

# Why a novel test?

- **IV regressions:** Andrews, Stock and Sun (2019): *'Screening on the first-stage F-statistic appears to compound, rather than reduce, inferential problems arising from weak instruments'*
- Table 7 from Zivot, Startz and Nelson (1998, IER):

| | Wald (90% nominal) |
|---|---|
| P(cover) | 0.801 |
| P(Test>cv) | 0.267 |
| P(cover \| Test>cv) | 0.445 |

IV regression, $T = 100$, $Cov(x_t, z_t) = 0.1$

Test = **pre-test statistic** for significance of first-stage regression

# Our testing approach

- Main properties:
  - **no pre-testing issues** ⇒ screening **does not affect** our strategy
  - computational straightforward ⇒ **normality test**
  - **robust** to $\alpha$-mixing VAR innovations and proxies (alternative to robust first-stage F-test)
- **Bootstrap algorithm**: Jentsch and Lunford's (2019 AER, 2019WP) **MBB**

# Our testing approach (cont'd)

- **Bootstrap as diagnostic tool**
- **DSGE models:** Bårdsen and Fanelli (2015, JBES): evaluate long run and short run implications
- **State-space models:** Angelini, Cavaliere and Fanelli (2020, WP) evaluate validity of standard regularity conditions
- **IV regressions:** Zhan (2018, WP): 'Detecting weak identification by bootstrap'
- **GMM:** Caner (2011): 'A prestest to differentiate between weak and nearly-weak instrument asymptotics'

# Sketch of the idea

- The **indirect-MD** estimation approach is based on $H_0$: $v_t$ **strong** for $\tilde{\varepsilon}_{2,t}$
- We consider an estimator $\hat{\theta}_T = f(\hat{\lambda}, \hat{\beta}_2)$ that **captures the strenght of the proxies**: $\lambda$ elements of $\Lambda$, $\beta_2$ free element of $\tilde{B}_2$: $\frac{\partial Y_t}{\partial \tilde{\varepsilon}'_{2,t}} = \tilde{B}_2$.
- Bootstrap counterpart: $\hat{\theta}^*_T = f(\hat{\lambda}^*, \hat{\beta}^*_2)$.
- Investigate **asymptotic behavior** of $\Gamma_T := T^{1/2} V_\theta^{-1/2} \left(\hat{\theta}_T - \theta_0\right)$ and $\Gamma^*_T := T^{1/2} \hat{V}_\theta^{-1/2} \left(\hat{\theta}^*_T - \hat{\theta}_T\right)$ under **strong** and **weak** proxies.

# Mechanics (part)

PROPOSITION 3. Under Assumptions 1-4

(i) **strong proxies**:

$$\Gamma_T \xrightarrow{d} \mathcal{N}(0, I) \quad , \quad \Gamma_T^* \xrightarrow{d^*}_p \mathcal{N}(0, I)$$

(ii) **weak proxies**:

$$\Gamma_T \xrightarrow{d} \mathcal{L} \neq \mathcal{N}(0, I)$$

cdf of $\Gamma_T^*$ is **stochastic in the limit**, hence it is not $\mathcal{N}(0, I)$

in the sense of Cavaliere and Georgiev (2020, ECMA).

# Operational implementation

- Simplest **operational form**:
  - Use **MBB** algorithm to generate $\left\{\hat{\theta}^*_{T:1}, \hat{\theta}^*_{T:2}, ..., \hat{\theta}^*_{T:N}\right\}$
  - $N$ selected such that $N/T \to 0$: Practical rule: **N:=T**$^{1/2}$;
  - Apply **normality test,** $\tau^*_v := DH(\hat{\theta}^*_{T:1}, \hat{\theta}^*_{T:2}, ..., \hat{\theta}^*_{T:N})$;
  - If normality rejected, $\tau^*_v > cv$, **proxies are weak**

# Multipliers

## 68%-MBB CI and projected S-region

direct

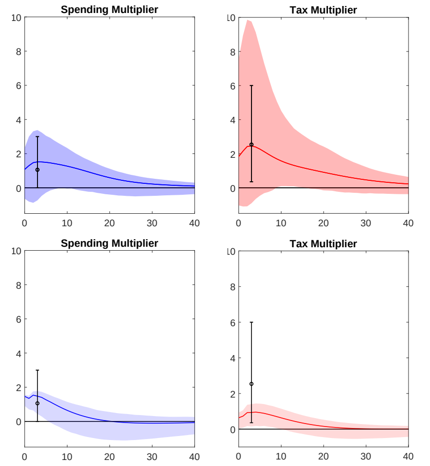
| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 1.08 | 1.84 |
| $(-0.64;2.34)$ | $(-1.03;7.58)$ |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.50[3] | 2.46[3] |
| $(-0.73;3.38)$ | $(-0.91;9.76)$ |

direct A&R projected

| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 0.74 | 1.91 |
| (0.00;3.00) | (0.22;6.00) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.06[3] | 2.55[3] |
| (0.00;3.00) | (0.37;6.00) |

indirect-MD

| $\mathcal{M}g_0$ | $\mathcal{M}tr_0$ |
|---|---|
| 1.47 | 0.64 |
| (0.90;1.56) | (0.04;0.93) |
| $\mathcal{M}g$ | $\mathcal{M}tr$ |
| 1.54[2] | 0.96[4] |
| (0.64;1.76) | (0.18;1.44) |




| $p$-val |
|---|
| 0.003 |

| $p$-val |
|---|
| 0.822 |

# Monte Carlo: rejection frequencies

$N = T^{1/2}$

| $\theta$ | $DH$ | $KS$ | $DH$ | $KS$ |
|---|---|---|---|---|
| **Strong proxy,** $corr(v_t, \varepsilon_{2,t}) \approx 59\%$ | | | | |
| | $T = 250$ | | $T = 1000$ | |
| | $corr = 59\%$ | | $corr = 59\%$ | |
| $\theta$ | $DH$ | $KS$ | $DH$ | $KS$ |
| $\beta_{2,1}$ | | 0.06(0.06) | | 0.05(0.06) |
| $\beta_{2,2}$ | 0.05(0.05) | 0.06(0.06) | 0.05(0.05) | 0.05(0.05) |
| $\beta_{2,3}$ | | 0.05(0.05) | | 0.05(0.05) |
| $\lambda$ | | 0.05(0.05) | | 0.05(0.05) |
| **Moderately weak proxy,** $corr(v_t, \varepsilon_{2,t}) \approx$ *mild* | | | | |
| | $T = 250$ | | $T = 1000$ | |
| | $corr = 25\%$ | | $corr = 13\%$ | |
| $\theta$ | $DH$ | $KS$ | $DH$ | $KS$ |
| $\beta_{2,1}$ | | 0.21(0.24) | | 0.36(0.36) |
| $\beta_{2,2}$ | 0.22(0.20) | 0.27(0.30) | 0.80(0.64) | 0.38(0.39) |
| $\beta_{2,3}$ | | 0.20(0.24) | | 0.30(0.33) |
| $\lambda$ | | 0.09(0.08) | | 0.10(0.11) |
| **Weak proxy,** $corr(v_t, \varepsilon_{2,t}) \approx$ *low* | | | | |
| | $T = 250$ | | $T = 1000$ | |
| | $corr = 5\%$ | | $corr = 2\%$ | |
| $\theta$ | $DH$ | $KS$ | $DH$ | $KS$ |
| $\beta_{2,1}$ | | 0.80(0.79) | | 0.93(0.93) |
| $\beta_{2,2}$ | 0.72(0.71) | 0.85(0.85) | 0.98(0.98) | 0.95(0.96) |
| $\beta_{2,3}$ | | 0.82(0.81) | | 0.95(0.95) |
| $\lambda$ | | 0.24(0.24) | | 0.50(0.49) |

# Screening

- Let $\rho_T$ **any statistic** computed on the proxy-SVAR, e.g.:

$$H_0 : IRF_{i,j}(h) = \gamma_{i,j,0} \ , \ \ \rho_T := \frac{\hat{\gamma}_{i,j} - \gamma_{i,j,0}}{\hat{V}_{i,j}^{1/2}}$$

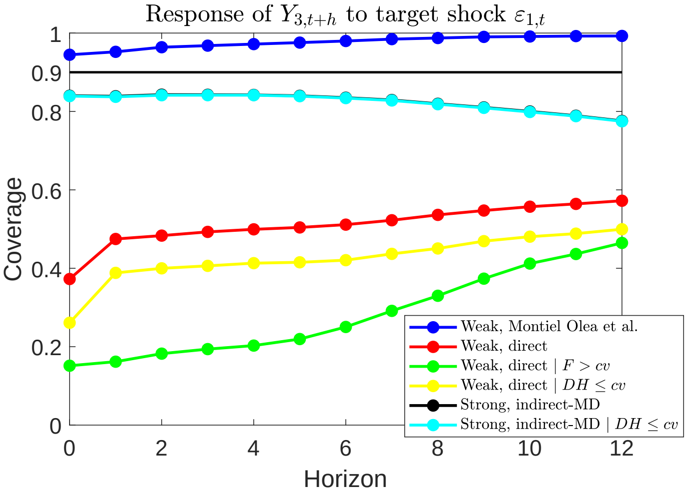
PROPOSITION 4. For any $x, c \in \mathbb{R}$, $T, N \to \infty$

$$P\left(\{\rho_T \leq x\} \cap \{\tau_v^* \leq c\}\right) - P(\rho_T \leq x)P(\tau_v^* \leq c) \longrightarrow 0$$

where $P(\cdot)$ defined on properly constructed probability space.

# IRF Coverages & screening

$T = 250$



# FISCAL MULTIPLIERS

# US fiscal multipliers

- VAR $Y_t := (TAX_t, G_t, GDP_t, RR_t)'$, sample: 1950Q1-2006Q4
- **Direct** approach: $z_t := (z_t^g, z_t^{tax})'$ for $\varepsilon_{1,t} := (\varepsilon_t^g, \varepsilon_t^{tax})'$
- **Indirect** approach: $v_t := (v_t^{tfp}, z_t^{rr})'$ for $\varepsilon_{2,t} := (\varepsilon_t^y, \varepsilon_t^{mp})'$

| | Direct | Proj. S-region | indirect-MD |
|---|---|---|---|
| peak $M_g$ | 1.50[3] (-0.73,3.38) | 1.07[3] (0,3) | 1.54[2] (0.64,1.8) |
| peak $M_{tax}$ | 2.46[3] (-0.91,9,76) | 2.53[3] (0.33,6) | 0.96[4] (0.2,1.4) |
| $\psi_{tax,y}$ | 3.48 (0.06,4,8) | | 2.06 (1.64, 2.49) |
| # restrictions | 2 | 6 | 1 |
| $\tau^*$ | | $z_t$ for $\varepsilon_{1,t}$ p-val=0.003 | $v_t$ for $\varepsilon_{2,t}$ p-val=0.82 |

68%-MBB (Hall) CI & projected 68% confidence regions

# Concluding remarks

- Proxy-SVARs, multiple shocks, with weak proxies

Contributions:
- Identification strategy + MD estimation
- Novel bootstrap diagnostic test
- Inference on US fiscal multipliers

- **Advantage**: $k > 1$, multiple target shocks
- **Limit**: the availability of (strong) **proxies for the auxiliary shocks** $\varepsilon_{2,t}$.

# Thank you

- THANK YOU

# Rationale of normality test

- Take $\Gamma^*_{1,T}$ e.g. first component of $\Gamma^*_T := T^{1/2}\hat{V}_\theta^{-1/2}\left(\hat{\theta}^*_T - \hat{\theta}_T\right)$;
- cdf $F^*_T(x) = P(\Gamma^*_{1,T} \leq x | \mathcal{D}_T)$ approximates the cdf $F_T(x) = P(\Gamma_{1,T} \leq x)$
- **Idea:** for any $x \in \mathbb{R}$ we want to evaluate **the distance**

$$F^*_T(x) - \mathcal{F}_{Gaus}(x)$$

- Under **strong proxies**, $\sup_{x\in\mathbb{R}} |F^*_T(x) - \mathcal{F}_{Gaus}(x)| \to_p 0$ as $T \to \infty$
- Since $F^*_T(x)$ is unknown, we use

$$F^*_{T,N}(x) := \frac{1}{N}\sum_{b=1}^{N} \mathbb{I}\left\{\Gamma^*_{1,T:b} \leq x\right\}.$$

# Rationale of bootstrap test (cont'd)

- **Normalized distance:**

$$\tau_v^*(x) := N^{1/2}\hat{V}_T(x)^{-1/2}(F_{T,N}^*(x) - \mathcal{F}_{Gaus}(x))$$

$$= \underbrace{N^{1/2}\hat{V}_T(x)^{-1/2}(F_{T,N}^*(x) - F_T^*(x))}_{\text{first term}} + \underbrace{N^{1/2}\hat{V}_T(x)^{-1/2}(F_T^*(x) - \mathcal{F}_{Gaus}(x))}_{\text{second term}}$$

  where $\hat{V}_T(x) := F_{T,N}^*(x)(1 - F_{T,N}^*(x))$ or $\hat{V}_T(x) := \mathcal{F}_{Gaus}(x)(1 - \mathcal{F}_{Gaus}(x))$.
- First term: **Gaussian** by CLT as $N \to \infty$ (any $T$) **regardless of strenght of proxies**
- Second term: **depends on the strength of the proxies**

# Rationale: focus on the second term

- Under **strong proxies** $\sup_{x\in\mathbb{R}} |F_T^*(x) - \mathcal{F}_{Gaus}(x)| \to_p 0$, assume it is also valid **standard Edgeworth expansion**

$$F_T^*(x) - \mathcal{F}_{Gaus}(x) = O_p(T^{-1/2})$$

as in Bose (1988, Annals) and Kilian (1998, REStat).
- Horowitz (2001, Handbook): expansion above is the typical case **in the presence of asymptotic normal statistic!**
- Then $N^{1/2}\hat{V}_T(x)^{-1/2}(F_T^*(x) - \mathcal{F}_{Gaus}(x)) = O_p(N^{1/2}T^{-1/2})$ **vanishes** as $T, N \to \infty$, **provided** $N = o(T)$
- Conversely, under **weak proxies** $N^{1/2}\hat{V}_T(x)^{-1/2}(F_T^*(x) - \mathcal{F}_{Gaus}(x))$ **diverges at the rate** $N^{1/2}$.

- Summing up, take normalized distance

$$\tau_v^*(x) = N^{1/2}\hat{V}_T(x)^{-1/2}(F_{T,N}^*(x) - F_T^*(x)) + N^{1/2}\hat{V}_T(x)^{-1/2}(F_T^*(x) - \mathcal{F}_{Gaus}(x))$$

- for $T, N \to \infty$ and $N/T = o(1)$ we have:
- $\tau_v^*(x)$ **Gaussian under strong proxies**
- $\tau_v^*(x)$ **deviates from Gaussian under weak proxies**
- Deviations from normality expected in the proxy-SVAR only with weak proxies.