

An Identification Strategy for Proxy-SVARs with Weak Proxies

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Motivations

- ▶ **Proxy-SVARs (SVAR-IVs)** have become standard tools to track dynamic causal effects [**IRFs**] in empirical macroeconomics: Mertens and Ravn (2013 AER); Stock and Watson (2018 EJ).
- ▶ **Main challenge:** external variables/proxies/instruments can be **weak**, i.e. poorly correlated with the **target structural shocks**, $\varepsilon_{1,t}$.
$$\text{Cov}(z_t, \varepsilon_{1,t}) = \frac{C}{T^{1/2}}$$
 à la Staiger and Stock (1997, ECMA).
- ▶ **Proxy-SVARs with weak proxies:** similar to IV regressions with weak instruments
⇒ asymptotic inference **is non-standard**.

Proxy-SVARs with weak proxies

- ▶ **Frequentist framework:**

Montiel-Olea, Stock and Watson (2020, JoE) **extend the logic of Anderson-Rubin tests** to proxy-SVARs.

- ▶ **Bayesian framework:**

Giacomini, Kitagawa and Read (2020, JoE), set-identification approach.

Proxy-SVARs with weak proxies (cont'd)

- ▶ Montiel-Olea, Stock and Watson (2020, JoE) approach:
- ▶ **Manageable** with **one target shock**, i.e. $\varepsilon_{1,t}$ scalar, $k = 1$.
- ▶ **Problematic** with **multiple target shocks**, i.e. $\varepsilon_{1,t}$ is $k \times 1$, $k > 1$, e.g. $\varepsilon_{1,t} := (\varepsilon_t^{tax}, \varepsilon_t^g)'$:
 - ▶ at least $k^2 > \frac{1}{2}k(k-1)$ restrictions needed to build S-regions;
 - ▶ **conservative** inference.

How we attack the problem

- ▶ Standard '**direct**' approach:

$$\varepsilon_t := \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \begin{matrix} k \times 1 & \text{target shocks} \\ (n - k) \times 1 & \text{auxiliary shocks} \end{matrix}$$

$$\text{proxies: } z_t = \Phi \varepsilon_{1,t} + \omega_{z,t}, \omega_{z,t} \perp \varepsilon_t, \text{rank}[\Phi_0] = ?$$

- ▶ We might have proxies v_t for $s \leq n - k$ **auxiliary shocks** $\tilde{\varepsilon}_{2,t}$:

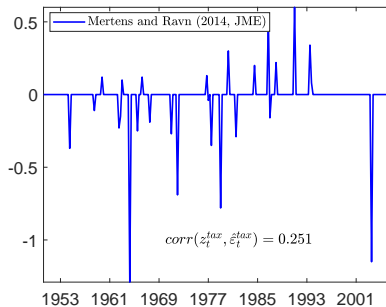
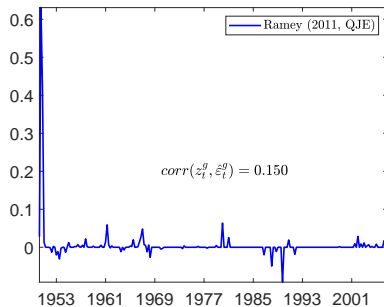
$$v_t = \Lambda \tilde{\varepsilon}_{2,t} + \omega_{v,t}, \omega_{v,t} \perp \varepsilon_t, \text{rank}[\Lambda_0] = s$$

- ▶ **Objective:** use **strong** proxies v_t for $\tilde{\varepsilon}_{2,t}$ **to recover the dynamic causal effects of the target shocks** $\varepsilon_{1,t}$.
- ▶ **Advantage:** circumvent non-standard methods of inference when z_t is suspected to be **weak** for $\varepsilon_{1,t}$, especially when $k > 1$.

Contributions

- ▶ **Identification strategy: indirect approach**
 - ▶ **B-form** \longleftrightarrow **A-form** representation of proxy-SVAR and design a **MD estimation** approach based on the A-form
 - ▶ derive **necessary and sufficient rank condition** for identification
 - ▶ \Rightarrow **standard asymptotics** if v_t is **strong** for $\tilde{\epsilon}_{2,t}$.
- ▶ **Diagnostic test** for **instrument relevance** v_t **based on bootstrap resampling - MBB** \Rightarrow **no pre-testing issues.**
- ▶ **Empirical:** focus in the **size and uncertainty** of US fiscal multipliers from a fiscal proxy-SVAR.
Ideal case, **k=2**, and narrative proxies for fiscal shocks suspected to be **weak**.

Fiscal proxies



Outline

- ▶ Identification strategy
 - ▶ B-form and A-form of proxy-SVARs
 - ▶ Indirect approach: MD estimation + asymptotic results
 - ▶ Example: tracking fiscal multipliers
- ▶ Bootstrap diagnostic test
 - ▶ Sketch of the idea
 - ▶ Mechanics (part)
 - ▶ Operational form
 - ▶ Monte Carlo
 - ▶ Screening on τ_v^*
- ▶ Fiscal multipliers from a quarterly fiscal proxy-SVAR
- ▶ Concluding remarks

IDENTIFICATION STRATEGY

B-form in proxy-SVARs

- ▶ SVAR: $Y_t - \Pi_1 Y_{t-1} = u_t$, $u_t = B\varepsilon_t$, $E(\varepsilon_t \varepsilon_t') = I_n$
- ▶ Proxy-SVAR:

$$\varepsilon_t := \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \quad \begin{array}{l} \text{target shocks} \quad k \times 1 \\ \text{auxiliary shocks} \quad (n - k) \times 1 \end{array}$$

$$u_t = B\varepsilon_t = B_1 \underbrace{\varepsilon_{1,t}}_{\text{target}} + B_2 \underbrace{\varepsilon_{2,t}}_{\text{auxiliary}}$$

- ▶ **Target:** $IRF_{i,j}(h) = \gamma_{i,j}(h) = e_i' (\Pi_1)^h B_1 e_j$, $j = 1, \dots, k$

A-form in proxy-SVARs

- ▶ A-form:

$$Au_t = \varepsilon_t \equiv \begin{cases} A'_1 u_t = \varepsilon_{1,t} & k \times 1 \\ A'_2 u_t = \varepsilon_{2,t} & (n-k) \times 1 \end{cases}$$

- ▶ focus on the k structural equations:

$$A'_1 u_t = (A'_{11} : A'_{12}) \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix} = A'_{11} u_{1,t} + A'_{12} u_{2,t} = \varepsilon_{1,t}$$

- ▶ **link:** $B_1 = \Sigma_u A_1 \Rightarrow \gamma_{i,j}(h) = e'_i (\Pi_1)^h B_1 e_j$, $j = 1, \dots, k$
- ▶ **Auxiliary shocks** have a role in the identification of A_1 and in recovering the **target shocks** !

Strategy induced by A-form

- ▶ Take:

$$A'_{11}u_{1,t} + A'_{12}u_{2,t} = \varepsilon_{1,t}$$

- ▶ Assume that A'_{11} ($k \times k$) is invertible (**not needed in our framework**):

$$u_{1,t} = \Psi u_{2,t} + (A'_{11})^{-1} \varepsilon_{1,t} \quad , \quad \Psi := - (A'_{11})^{-1} A'_{12}$$

and recall that

$$u_{2,t} = B_{21}\varepsilon_{1,t} + B_{22}\varepsilon_{2,t}$$

- ▶ \Rightarrow we might consider proxies v_t for the auxiliary shocks $\varepsilon_{2,t}$!
- ▶ With $k > 1$, A'_{11} must be triangular: $\frac{1}{2}k(k-1)$ restrictions in Mertens and Ravn's (2013, AER) in their IV approach.
Not necessary in our framework.

Indirect-MD approach: assumptions and definition

- ▶ ASSUMPTION 1: DGP ∈ VAR, Π_1 stable;
- ▶ ASSUMPTION 2: $u_t \sim \alpha$ -mixing;
- ▶ ASSUMPTION 3: $A = B^{-1} \Rightarrow \text{rank}[A'_1] = k = \text{rank}[B_1]$
- ▶ ASSUMPTION 4: given $\tilde{\varepsilon}_{2,t}$ subset of $\varepsilon_{2,t}$,
 $\text{dim}(\tilde{\varepsilon}_{2,t}) = s \leq n - k$

$$v_t = \Lambda \tilde{\varepsilon}_{2,t} + \omega_{v,t} \quad , \quad \omega_{v,t} \perp \varepsilon_t$$

- ▶ DEFINITION: v_t is:
 - ▶ **strong** for $\tilde{\varepsilon}_{2,t}$ if $\text{rank}[\Lambda] = s$ in \mathcal{N}_{λ_0} regardless of T ;
 - ▶ **weak** for $\tilde{\varepsilon}_{2,t}$ if $\Lambda = \Lambda_1 + T^{-1/2}C$, $C \neq 0_{s \times s}$,
 $\text{rank}[\Lambda_1] < s$ in \mathcal{N}_{λ_0} .

Indirect-MD approach: moment conditions

- ▶ Recall: **estimate** $\gamma_{i,j}(h) = e_i' (\Pi_1)^h B_1 e_j$, $j = 1, \dots, k$ by **exploiting** $B_1 = \Sigma_u A_1$

- ▶ A-form:

$$A_1' u_t = \varepsilon_{1,t}$$

- ▶ Under Assumption 4, implied **moment conditions**:

$$\begin{aligned} A_1' \Sigma_u A_1 &= I_k & \frac{1}{2} k(k+1) \times 1 \\ A_1' \Sigma_{u,v} &= 0_{ks \times 1} & ks \times 1 \end{aligned}$$

- ▶ $k > 1$, at least $\frac{1}{2} k(k-1)$ restrictions on A_1' :

$$\text{vec}(A_1') = S_{A_1} \alpha + s_{A_1}$$

Indirect-MD approach: distance function

- Summarize moment conditions and restrictions in the **distance function**:

$$g(\sigma_0^+, \alpha_0) := \text{vec} \begin{pmatrix} A_1' \Sigma_u A_1 - I_k \\ A_1' \Sigma_{u,v} \end{pmatrix} = 0_{m \times 1}, \quad A_1 := A_1(\alpha).$$

- $\hat{\sigma}_T^+ := (\text{vec}(\hat{\Sigma}_{u,v})', \text{vech}(\hat{\Sigma}_u)')'$ easily recoverable from

$$\begin{pmatrix} Y_t \\ v_t \end{pmatrix} = \begin{pmatrix} \Pi_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ v_{t-1} \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}$$

$$\Sigma_\eta := E \left\{ \begin{pmatrix} u_t \\ v_t \end{pmatrix} (u_t', v_t') \right\} = \begin{pmatrix} \Sigma_u & \Sigma_{u,v} \\ \Sigma_{v,u} & \Sigma_v \end{pmatrix}$$

consistent and **asympt. Gaussian** under Ass. 1-2 and 4.

Indirect-MD approach: estimation

- ▶ Given $g(\sigma_0^+, \alpha_0) = 0_{m \times 1}$
- ▶ Then

$$\hat{\alpha}_T := \arg \min_{\alpha \in \mathcal{I}_\alpha} \hat{g}_T(\hat{\sigma}_T^+, \alpha)' \hat{V}_{gg}(\bar{\alpha})^{-1} \hat{g}_T(\hat{\sigma}_T^+, \alpha)$$

$$\hat{V}_{gg}(\bar{\alpha}) := G_{\sigma^+}(\sigma^+, \alpha) \hat{V}_{\sigma^+} G_{\sigma^+}(\sigma^+, \alpha)',$$

$$G_{\sigma^+}(\sigma^+, \alpha) := \frac{\partial g(\sigma^+, \alpha)}{\partial \sigma^+} \text{ Jacobian,}$$

$$V_{\sigma^+} := \lim_{T \rightarrow \infty} \text{Var}(T^{1/2}(\hat{\sigma}_T^+ - \sigma_0^+))$$

$\bar{\alpha}$ preliminary estimate of α

- ▶ Newey and McFadden (1994, Handbook, Ch.36)

Indirect-MD approach: identification

PROPOSITION 1 [RANK CONDITION] Under Assumptions 1-4, the MD objective function has **unique** minimum in \mathcal{N}_{α_0} **iff**:

$$\text{rank} \left[\begin{pmatrix} 2D_k^+(A_1' \Sigma_u \otimes I_k) \\ (\Sigma_{v,u} \otimes I_k) \end{pmatrix} S_{A_1} \right] = a, \quad a := \dim(\alpha).$$

Necessary order condition:

$$\ell \geq \frac{1}{2}k(k-1) \Leftrightarrow a \leq \frac{1}{2}k(k+1) + ks$$

COROLLARY 1: If rank condition holds $\Rightarrow \text{rank}[\Lambda_0] = s$ in \mathcal{N}_{λ_0} ,
i.e. v_t **strong** for $\tilde{\varepsilon}_{2,t}$.

Indirect-MD approach: asymptotic properties

PROPOSITION 2. Under Assumptions 1-4, **rank condition holds**, $\alpha_0 \in \text{int}(\mathcal{T}_\alpha)$, as $T \rightarrow \infty$:

(i) $(\hat{\alpha}_T - \alpha_0) \xrightarrow{p} 0_{a \times 1}$

(ii) $T^{1/2} (\hat{\alpha}_T - \alpha_0) \xrightarrow{d} \mathcal{N}(0, V_\alpha)$, $V_\alpha := \{G_\alpha' V_{gg}(\bar{\alpha})^{-1} G_\alpha\}^{-1}$

- Implications on IRFs: because of $B_1 = \Sigma_u A_1$

$$T^{1/2} (\hat{\gamma}_{i,j}(h) - \gamma_{i,j,0}(h)) \xrightarrow{d} \mathcal{N}(0, V_{\gamma_{ij}}), \quad h = 0, \dots, h_{\max}$$

- **Inference is standard**

Indirect-MD approach: qualifications on the methodology

- ▶ **A-form** of proxy-SVARs: typical of the Bayesian approach: Caldara and Herbst (2019, AEJ-Macro), Arias, Rubio-Ramirez and Waggoner (2020, JoE), Giacomini, Kitagawa and Read (2020, JoE):
 - ▶ **proxies are used to instrument** the **target shocks**, not the **auxiliary** ones
- ▶ Caldara and Kamps (2017, REStud), fiscal proxy-SVAR, use **non-fiscal proxies** to instrument **non-fiscal shocks**, **Bayesian - penalty function - approach**.
- ▶ Our: **frequentist approach**, **point-identification**, **does not necessarily require** a policy reaction function.
- ▶ When **$k > 1$ MD** estimation more general than **IV**, does not require **Choleski-type restrictions**, see e.g. Mertens and Ravn (2013, AER).

Example: tracking fiscal multipliers from a fiscal proxy-SVAR

- ▶ VAR: $n = 4$

$$Y_t = \begin{pmatrix} TAX_t \\ G_t \\ DGP_t \\ RR_t \end{pmatrix} \begin{array}{l} \text{tax revenues} \\ \text{fiscal spending} \\ \text{real GDP} \\ \text{real interest rate} \end{array}$$

- ▶ **Fiscal multipliers:** $k = 2$, $h = 0, 1, \dots$

$$\mathbb{M}_{h,tax} := \frac{\left(\frac{\partial DGP_{t+h}}{\partial \varepsilon_t^{tax}} \right)}{\beta_{tax,tax}} \times \text{scale}, \quad \mathbb{M}_{h,g} := \frac{\left(\frac{\partial DGP_{t+h}}{\partial \varepsilon_t^g} \right)}{\beta_{g,g}} \times \text{scale}$$

Blanchard and Perotti (2002, QJE);

Mertens and Ravn (2014, JME);

Caldara and Kamps (2017, REStud).

Example: fiscal multipliers, B-form, direct approach

- ▶ B-form: $k = 2, (n - k = 2)$

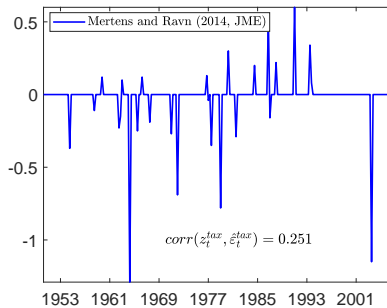
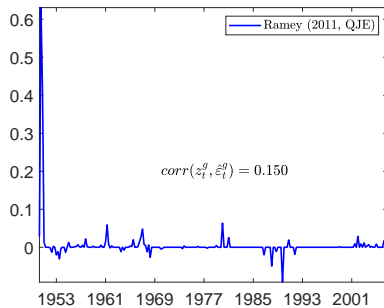
$$\begin{pmatrix} u_t^{tax} \\ u_t^g \\ u_t^y \\ u_t^{rr} \end{pmatrix} = \underbrace{\begin{pmatrix} \beta_{tax,tax} & \beta_{tax,g} \\ \beta_{g,tax} & \beta_{g,g} \\ \beta_{y,tax} & \beta_{y,g} \\ \beta_{rr,tax} & \beta_{rr,g} \end{pmatrix}}_{B_1} \underbrace{\begin{pmatrix} \varepsilon_t^{tax} \\ \varepsilon_t^g \end{pmatrix}}_{\text{target } \varepsilon_{1,t}} + B_2 \underbrace{\begin{pmatrix} \varepsilon_t^y \\ \varepsilon_t^{mp} \end{pmatrix}}_{\text{auxiliary } \varepsilon_{2,t}}$$

- ▶ Proxies:

$$\begin{pmatrix} z_t^{tax} \\ z_t^g \\ z_t \end{pmatrix} = \underbrace{\begin{pmatrix} \varphi_{11} & 0 \\ 0 & \varphi_{22} \end{pmatrix}}_{\Phi} \begin{pmatrix} \varepsilon_t^{tax} \\ \varepsilon_t^g \end{pmatrix}_{\varepsilon_{1,t}} + \begin{pmatrix} \omega_t^{tax} \\ \omega_t^g \end{pmatrix}_{\omega_{z,t}}$$

- ▶ $E(u_t z_t') = \Sigma_{u,z} = B_1 \Phi'$.

Fiscal proxies



Multipliers

68%-MBB CI and projected S-region

direct

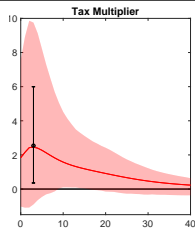
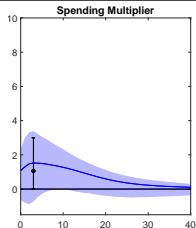
$\mathcal{M}g_0$	$\mathcal{M}tr_0$
1.08	1.84
(-0.64;2.34)	(-1.03;7.58)
$\mathcal{M}g$	$\mathcal{M}tr$
1.50[3]	2.46[3]
(-0.73;3.38)	(-0.91;9.76)

direct A&R projected

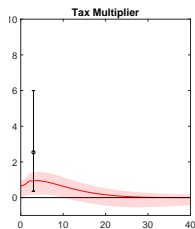
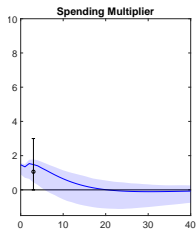
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indirect-MD

$\mathcal{M}g_0$	$\mathcal{M}tr_0$
1.47	0.64
(0.90;1.56)	(0.04;0.93)
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1.54[2]	0.96[4]
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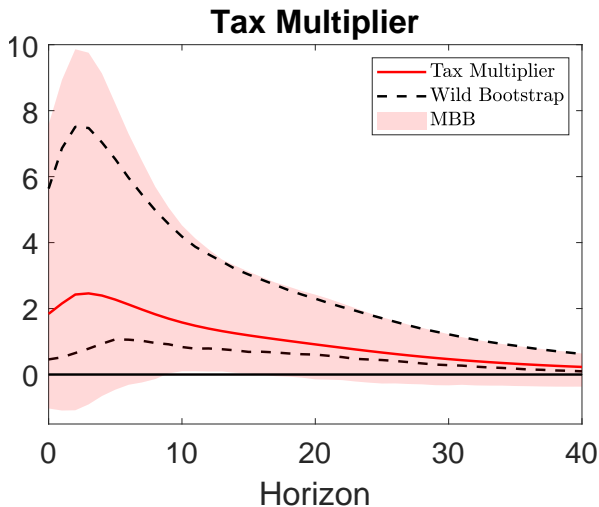


p -val
0.003



p -val
0.822

MBB vs Wild Bootstrap



Example: Identification-robust method

- ▶ Proxies z_t are suspected to be **weak**:

$$\Sigma_{u,z} = B_1 \Phi' \Rightarrow \begin{matrix} k \times k \\ k \times (n-k) \end{matrix} \begin{pmatrix} \Sigma_{u1,z} \\ \Sigma_{u2,z} \end{pmatrix} = \begin{pmatrix} B_{11} \Phi' \\ B_{21} \Phi' \end{pmatrix}$$

- ▶ With $B_{11} = I_k$ ($k^2 = 4$ restrictions)
- ▶ solve Φ (regardless of rank) as function of reduced form parameters:
- ▶ $\Rightarrow \Phi' = \Sigma_{u1,z} \Rightarrow \Sigma_{u2,z} - B_{21} \Sigma_{u1,z} = 0_{k(n-k)}$.

Example: Identification-robust method (cont'd)

- ▶ Null hypothesis (at given horizon h)

$$H_0 : \left(\frac{\partial GDP_{t+h}}{\partial \varepsilon_t^{tax}}, \frac{\partial GDP_{t+h}}{\partial \varepsilon_t^g} \right) = e_3'(\Pi_1)^h B_1 = (a_{tax}, b_g)$$

implies:

$$e_3'(\Pi_1)^h \Sigma_{u,z} - (a_{tax}, b_g)(I_k \vdots 0) \Sigma_{u,z} = (0, 0)$$

$$\Rightarrow Rvec(\Sigma_{u,z}) = 0_{2 \times 1}, \quad R := R(\Pi_1, a_{tax}, b_g)$$

- ▶ **S-region** with asymptotic coverage $\geq 1 - \varrho$:

$$\left\{ (a_{tax}, b_g), T(Rvec(\hat{\Sigma}_{u,z}))' (\hat{V}_R)^{-1} (Rvec(\hat{\Sigma}_{u,z})) \leq \chi_{2,1-\varrho}^2 \right\}$$

- ▶ projection method for a_{tax} and b_g .

Multipliers

68%-MBB CI and projected S-region

direct

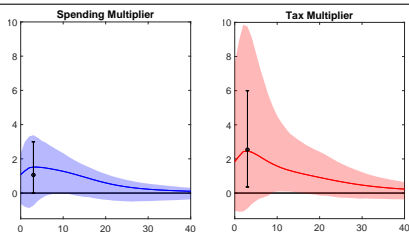
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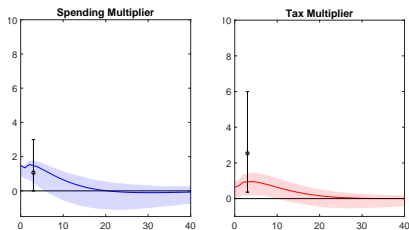
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Example: fiscal multipliers, A-form, indirect approach

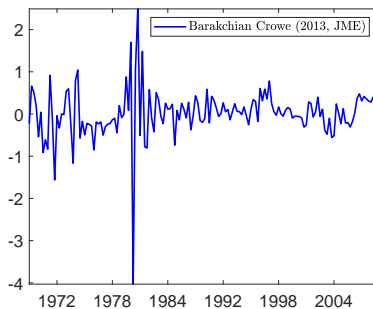
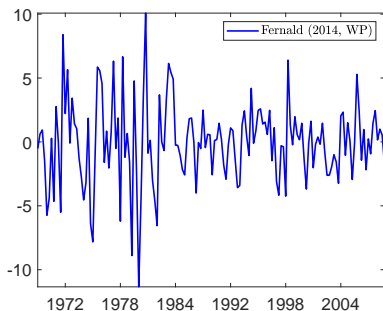
- ▶ A-form $k = 2$, $n - k = 2$

$$\begin{pmatrix} \alpha_{tax,tax} & \alpha_{tax,g} \\ \alpha_{g,tax} & \alpha_{g,g} \end{pmatrix} \begin{pmatrix} u_t^{tax} \\ u_t^g \end{pmatrix} \\ A'_{11} \quad u_{1,t} \\ + \begin{pmatrix} \alpha_{tax,y} & \alpha_{tax,r} \\ \alpha_{g,y} & \alpha_{g,r} \end{pmatrix} \begin{pmatrix} u_t^y \\ u_t^{rr} \end{pmatrix} = \begin{pmatrix} \varepsilon_t^{tax} \\ \varepsilon_t^g \end{pmatrix} \\ A'_{12} \quad u_{2,t} \quad \varepsilon_{1,t}$$

- ▶ $\frac{1}{2}k(k-1) = 1$ restrictions on $A'_1 \equiv (A'_{11} \vdots A'_{12})$ **consistent with rank condition**
- ▶ Proxies:

$$\begin{pmatrix} v_t^{tfp} \\ v_t^{mp} \end{pmatrix} = \Lambda \begin{pmatrix} \varepsilon_t^y \\ \varepsilon_t^{mp} \end{pmatrix} + \begin{pmatrix} \omega_t^{tfp} \\ \omega_t^{mp} \end{pmatrix}.$$

Indirect-MD: non-fiscal proxies for auxiliary shocks



Multipliers

68%-MBB CI and projected S-region

direct

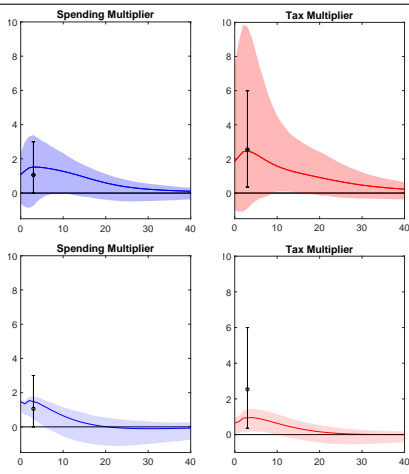
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BOOTSTRAP DIAGNOSTIC TEST FOR INSTRUMENT RELEVANCE

Why a novel test?

- ▶ **IV regressions:** Andrews, Stock and Sun (2019): *'Screening on the first-stage F-statistic appears to compound, rather than reduce, inferential problems arising from weak instruments'*
- ▶ Table 7 from Zivot, Startz and Nelson (1998, IER):

	Wald (90% nominal)
P(cover)	0.801
P(Test > cv)	0.267
P(cover Test > cv)	0.445

IV regression, $T = 100$, $Cov(x_t, z_t) = 0.1$

Test = **pre-test statistic** for significance of first-stage regression

Our testing approach

- ▶ **Bootstrap as diagnostic tool**
- ▶ Main properties:
 - ▶ **no pre-testing issues**
 - ▶ computational straightforward \Rightarrow **normality test**
 - ▶ **Robust** to α -mixing VAR innovations and proxies (alternative to robust first-stage F-test)
- ▶ **DSGE models:** Bårdsen and Fanelli (2015, JBES): evaluate long run and short run implications
- ▶ **State-space models:** Angelini, Cavaliere and Fanelli (2020, WP) evaluate validity of standard regularity conditions
- ▶ **IV regressions:** Zhan (2018, WP): 'Detecting weak identification by bootstrap'
- ▶ **GMM:** Caner (2011): 'A pretest to differentiate between weak and nearly-weak instrument asymptotics'

Sketch of the idea

- ▶ The **indirect-MD** estimation approach is based on $H_0: v_t$ **strong** for $\tilde{\varepsilon}_{2,t}$
- ▶ We consider an estimator $\hat{\theta}_T = f(\hat{\lambda}, \hat{\beta}_2)$ that **captures the strenght of the proxies**: λ elements of Λ , β_2 free element of $\tilde{B}_2: \frac{\partial Y_t}{\partial \tilde{\varepsilon}_{2,t}} = \tilde{B}_2$.
- ▶ Bootstrap counterpart: $\hat{\theta}_T^* = f(\hat{\lambda}^*, \hat{\beta}_2^*)$.
- ▶ Investigate **asymptotic behavior** of $\Gamma_T := T^{1/2} V_\theta^{-1/2} (\hat{\theta}_T - \theta_0)$ and $\Gamma_T^* := T^{1/2} \hat{V}_\theta^{-1/2} (\hat{\theta}_T^* - \hat{\theta}_T)$ under **strong** and **weak** proxies.

Mechanics (part)

PROPOSITION 3. Under Assumptions 1-4

(i) **strong proxies:**

$$\Gamma_T \xrightarrow{d} \mathcal{N}(0, I) \quad , \quad \Gamma_T^* \xrightarrow{d^*} \mathcal{N}(0, I)$$

(ii) **weak proxies:**

$$\Gamma_T \xrightarrow{d} \mathcal{L} \neq \mathcal{N}(0, I)$$

cdf of $\Gamma_T^* \xrightarrow{d^*}$ stochastic in the limit, hence it is not $\mathcal{N}(0, I)$

in the sense of Cavaliere and Georgiev (2020, ECMA).

Operational implementation

► Simplest **operational form**:

- Use **MBB** algorithm to generate $\{\hat{\theta}_{T:1}^*, \hat{\theta}_{T:2}^*, \dots, \hat{\theta}_{T:N}^*\}$
- N selected such that $N/T \rightarrow 0$: Practical rule: $N := T^{1/2}$;
- Apply **normality test**, $\tau_v^* := DH(\hat{\theta}_{T:1}^*, \hat{\theta}_{T:2}^*, \dots, \hat{\theta}_{T:N}^*)$;
- If normality rejected, $\tau_v^* > cv$, **proxies are weak**

Monte Carlo: rejection frequencies

$N = T^{1/2}$

Strong proxy, $\text{corr}(v_t, \varepsilon_{2,t}) \approx 59\%$				
$T = 250$		$T = 1000$		
$\text{corr} = 59\%$		$\text{corr} = 59\%$		
θ	DH	KS	DH	KS
$\beta_{2,1}$		0.06(0.06)		0.05(0.06)
$\beta_{2,2}$	0.05(0.05)	0.06(0.06)	0.05(0.05)	0.05(0.05)
$\beta_{2,3}$		0.05(0.05)		0.05(0.05)
λ		0.05(0.05)		0.05(0.05)

Moderately weak proxy, $\text{corr}(v_t, \varepsilon_{2,t}) \approx \text{mild}$				
$T = 250$		$T = 1000$		
$\text{corr} = 25\%$		$\text{corr} = 13\%$		
θ	DH	KS	DH	KS
$\beta_{2,1}$		0.21(0.24)		0.36(0.36)
$\beta_{2,2}$	0.22(0.20)	0.27(0.30)	0.80(0.64)	0.38(0.39)
$\beta_{2,3}$		0.20(0.24)		0.30(0.33)
λ		0.09(0.08)		0.10(0.11)

Weak proxy, $\text{corr}(z_t, \varepsilon_{1,t}) \approx \text{low}$				
$T = 250$		$T = 1000$		
$\text{corr} = 5\%$		$\text{corr} = 2\%$		
θ	DH	KS	DH	KS
$\beta_{2,1}$		0.80(0.79)		0.93(0.93)
$\beta_{2,2}$	0.72(0.71)	0.85(0.85)	0.98(0.98)	0.95(0.96)
$\beta_{2,3}$		0.82(0.81)		0.95(0.95)
λ		0.24(0.24)		0.50(0.49)

Screening

- ▶ Let ρ_T **any statistic** computed on the proxy-SVAR, e.g.:

$$H_0 : IRF_{i,j}(h) = \gamma_{i,j,0} , \quad \rho_T := \frac{\hat{\gamma}_{i,j} - \gamma_{i,j,0}}{\hat{V}_{i,j}^{1/2}}$$

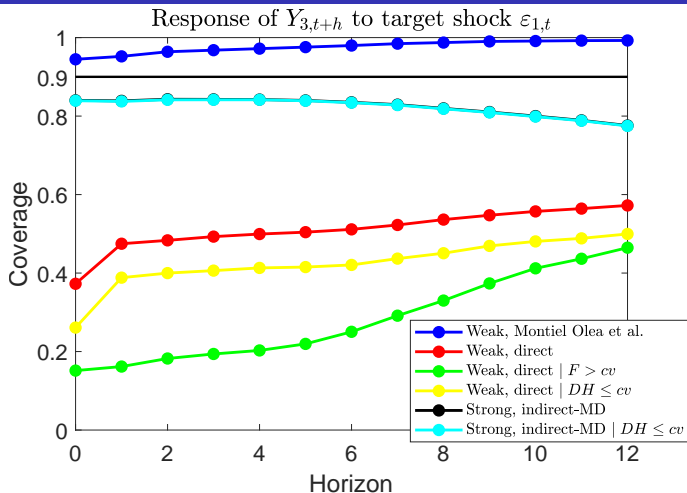
PROPOSITION 4. For any $x, c \in \mathbb{R}$, $T, N \rightarrow \infty$

$$P(\{\rho_T \leq x\} \cap \{\tau_v^* \leq c\}) - P(\rho_T \leq x)P(\tau_v^* \leq c) \longrightarrow 0$$

where $P(\cdot)$ defined on properly constructed probability space.

IRF Coverages & screening

$T = 250$



FISCAL MULTIPLIERS

US fiscal multipliers

- ▶ VAR $Y_t := (TAX_t, G_t, GDP_t, RR_t)'$, sample: 1950Q1-2006Q4
- ▶ **Direct** approach: $z_t := (z_t^g, z_t^{tax})'$ for $\varepsilon_{1,t} := (\varepsilon_t^g, \varepsilon_t^{tax})'$
- ▶ **Indirect** approach: $v_t := (v_t^{tfp}, z_t^{rr})'$ for $\varepsilon_{2,t} := (\varepsilon_t^y, \varepsilon_t^{mp})'$

	Direct	Proj. S-region	indirect-MD
peak M_g	1.50[3] (-0.73,3.38)	1.07[3] (0,3)	1.54[2] (0.64,1.8)
peak M_{tax}	2.46[3] (-0.91,9.76)	2.53[3] (0.33,6)	0.96[4] (0.2,1.4)
$\psi_{tax,y}$	3.48 (0.06,4.8)		2.06 (1.64, 2.49)
# restrictions	2	6	1
τ^*		Z_t for $\varepsilon_{1,t}$ p-val=0.003	V_t for $\varepsilon_{2,t}$ p-val=0.82

68%-MBB (Hall) CI & projected 68% confidence regions

Concluding remarks

- ▶ Proxy-SVARs, multiple shocks, with weak proxies

Contributions: $\left\{ \begin{array}{l} \text{Identification strategy + MD estimation} \\ \text{Novel bootstrap diagnostic test} \\ \text{Inference on US fiscal multipliers} \end{array} \right.$

- ▶ **Advantage:** $k > 1$, multiple target shocks
- ▶ **Limit:** the availability of (strong) **proxies for the auxiliary shocks** $\varepsilon_{2,t}$.

Thank you

▶ THANK YOU

Rationale of normality test

- ▶ Take $\Gamma_{1,T}^*$ e.g. first component of $\Gamma_T^* := T^{1/2} \hat{V}_\theta^{-1/2} (\hat{\theta}_T^* - \hat{\theta}_T)$;
- ▶ cdf $F_T^*(x) = P(\Gamma_{1,T}^* \leq x | \mathcal{D}_T)$ approximates the cdf $F_T(x) = P(\Gamma_{1,T} \leq x)$
- ▶ **Idea:** for any $x \in \mathbb{R}$ we want to evaluate **the distance**

$$F_T^*(x) - \mathcal{F}_{Gaus}(x)$$

- ▶ Under **strong proxies**, $\sup_{x \in \mathbb{R}} |F_T^*(x) - \mathcal{F}_{Gaus}(x)| \rightarrow_p 0$ as $T \rightarrow \infty$
- ▶ Since $F_T^*(x)$ is unknown, we use

$$F_{T,N}^*(x) := \frac{1}{N} \sum_{b=1}^N \mathbb{I} \{ \Gamma_{1,T:b}^* \leq x \} .$$

Rationale of bootstrap test (cont'd)

- ▶ **Normalized distance:**

$$\begin{aligned} \tau_v^*(x) &:= N^{1/2} \hat{V}_T(x)^{-1/2} (F_{T,N}^*(x) - \mathcal{F}_{Gaus}(x)) \\ &= \underbrace{N^{1/2} \hat{V}_T(x)^{-1/2} (F_{T,N}^*(x) - F_T^*(x))}_{\text{first term}} \\ &\quad + \underbrace{N^{1/2} \hat{V}_T(x)^{-1/2} (F_T^*(x) - \mathcal{F}_{Gaus}(x))}_{\text{second term}} \end{aligned}$$

where $\hat{V}_T(x) := F_{T,N}^*(x)(1 - F_{T,N}^*(x))$ or

$\hat{V}_T(x) := \mathcal{F}_{Gaus}(x)(1 - \mathcal{F}_{Gaus}(x))$.

- ▶ First term: **Gaussian** by CLT as $N \rightarrow \infty$ (any T) **regardless of strenght of proxies**
- ▶ Second term: **depends on the strength of the proxies**

Rationale: focus on the second term

- ▶ Under **strong proxies** $\sup_{x \in \mathbb{R}} |F_T^*(x) - \mathcal{F}_{Gaus}(x)| \rightarrow_p 0$, assume it is also valid **standard Edgeworth expansion**

$$F_T^*(x) - \mathcal{F}_{Gaus}(x) = O_p(T^{-1/2})$$

as in Bose (1988, Annals) and Kilian (1998, REStat).

- ▶ Horowitz (2001, Handbook): expansion above is the typical case **in the presence of asymptotic normal statistic!**
- ▶ Then $N^{1/2} \hat{V}_T(x)^{-1/2} (F_T^*(x) - \mathcal{F}_{Gaus}(x)) = O_p(N^{1/2} T^{-1/2})$ **vanishes** as $T, N \rightarrow \infty$, **provided** $N = o(T)$
- ▶ Conversely, under **weak proxies** $N^{1/2} \hat{V}_T(x)^{-1/2} (F_T^*(x) - \mathcal{F}_{Gaus}(x))$ **diverges at the rate** $N^{1/2}$.

- ▶ Summing up, take normalized distance

$$\begin{aligned}\tau_v^*(x) &= N^{1/2} \hat{V}_T(x)^{-1/2} (F_{T,N}^*(x) - F_T^*(x)) \\ &\quad + N^{1/2} \hat{V}_T(x)^{-1/2} (F_T^*(x) - \mathcal{F}_{Gaus}(x))\end{aligned}$$

- ▶ for $T, N \rightarrow \infty$ and $N/T = o(1)$ we have:
- ▶ $\tau_v^*(x)$ **Gaussian under strong proxies**
- ▶ $\tau_v^*(x)$ **deviates from Gaussian under weak proxies**
- ▶ Deviations from normality expected in the proxy-SVAR only with weak proxies.